5.13 HALL EFFECT

When a specimen (metal or semiconductor) carrying a current I is placed in a transverse magnetic field* B*, then a potential is developed in the specimen in the direction perpendicular to both I and* B*. The phenomenon is known as Hall effect. Hall effect was discovered in 1879.

Figure (19) shows a strip or bar of a material carrying a current I in the positive X-direction. Let the magnetic field **B** is applied in the positive Z-direction. The width and thickness of the bar are y and z respectively.

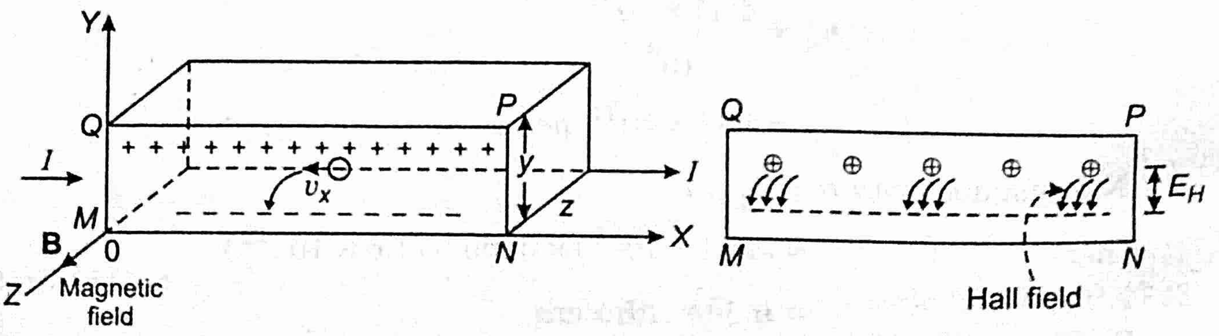

Fig. 19. A bar carrying a current *I* placed in magnetic field B.

The charge carriers (say electrons in present case) experience a deflecting force in downward direction, *i.e.*, in negative *Y*-direction. This causes negative charges to accumulate on the bottom face while positive ions are collected at upper face. This separation of charges sets up an electrostatic field inside the conductor in *Y*-direction. This is called ***Hall field*** E_H and the effect is called as ***Hall effect.***

5.13-1 HALL COEFFICIENT

The force on electron due to electric field $= -e\,E_H$

The force on electron due to magnetic field $= -e\,v_x\,B_z$

In equilibrium, $e\,E_H = e\,v_x\,B_z$ or $E_H = v_x\,B_z$ ...(1)

The current density J_x in *X*-direction is given by

$$J_x = -n\,e\,v_x \quad \text{or} \quad v_x = -J_x/n\,e \qquad ...(2)$$

where n is the charge carrier current density, *i.e.*, number of charge carriers per unit volume.

Substituting the value of v_x from eq. (2) in eq (1), we get

$$E_H = -\left(\frac{J_x}{n\,e}\right)B_z = -\left(\frac{1}{n\,e}\right)J_x\,B_z \qquad ...(3)$$

This equation shows that Hall field E_H is proportional, both to current density (or current) and to magne'ic field. The proportionality constant is usually denoted by R_H. Thus we have

$$\frac{E_H}{J_x\,B_z} = -\frac{1}{n\,e} = R_H \qquad ...(4)$$

R_H is known as Hall coefficient.

Let us express the Hall electric field E_H in terms of Hall potential difference V_H. Let y and z be the thickness and width of the bar along Y and *Z*-axes respectively.

$$E_H = \left(\frac{V_H}{y}\right)$$

From eq. (4), $$\frac{(V_H/y)}{J_x\,B_z} = R_H$$

or $$V_H = J_x\,B_z\,y\,R_H \qquad ...(5)$$

Further, $$J_x = (I_x/y\,z)$$

$\therefore$ $$V_H = \left(\frac{I_x}{y\,z}\right)B_z\,y\,R_H = \frac{I_x\,B_z\,R_H}{z}$$

Therefore, $$R_H = \frac{V_H\,z}{I_x\,B_z} \qquad ...(6)$$

Eq. (6) is another form of Hall Coefficient.

5.13-2 MOBILITY AND HALL ANGLE

The mobility is defined as drift velocity v_x acquired per unit electric field

Mobility, $\mu = v_x / E$ or $v_x = \mu E$ ...(7)

From eq. (1), $E_H = v_x B_z = \mu E B_z$ ...(8)

From eq. (3), $E_H = -\left(\frac{1}{ne}\right) J_x B_z = R_H J_x B_z$ ...(9)

Comparing eqs. (8) and (9), we get

$$R_H J_x B_z = \mu E B_z \quad \text{or} \quad \mu = \frac{R_H J_x}{E} = \sigma R_H \quad (\because \sigma = J_x / E)$$

where σ is the conductivity.

Therefore, mobility μ is given by

$$\mu = \sigma R_H \qquad ...(10)$$

From eq. (8), $E_H = \mu E B_z$

or $\mu = \frac{E_H}{E}\left(\frac{1}{B_z}\right) = \theta_H \cdot \left(\frac{1}{B_z}\right)$

where $\theta_H = \frac{E_H}{E}$ = Hall angle ...(11)

5.13-3 EXPERIMENTAL MEASUREMENT OF HALL COEFFICIENT

The arrangement for measurement of Hall voltage is shown in fig. (20).

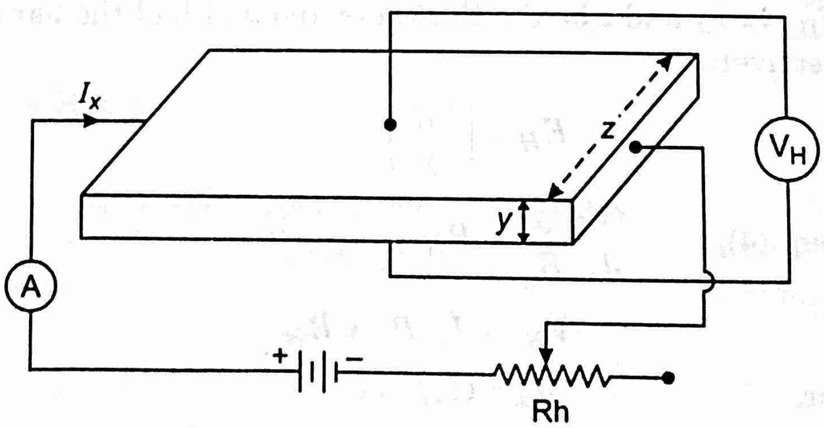

Fig. 20. Experimental arrangement of Hall voltage.

A rectangular slab of given material is taken. A current (say I_x) is passed in this slab along X-direction with the help of battery and rheostat Rh. The slab is placed between the pole pieces of an electromagnet such that magnetic field is in a perpendicular direction of current (say in Z-direction).

A voltmeter is connected in Y-direction as shown in figure. The voltmeter measures the Hall voltage V_H.

Further, current density $J = \frac{I_x}{A} = \frac{I_x}{yz}$

Importance of Hall Effect

Following are the importance of Hall effect :

1. The sign of charge carrier can be determined
2. Charge carrier concentration (n) can be determined
3. Mobility of charge carriers can be determined

5.14 EXPRESSION FOR HALL ANGLE

The net electric field E in a specimen is the vector sum of the following two components :

1. Electric field component E_x in X-direction due to flow of current I_x in X direction.
2. Electric field E_H due to Hall effect in Y-direction. The two components are shown in fig. (21).

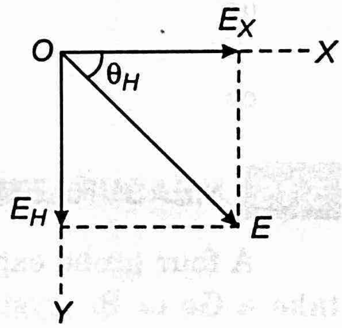

Fig. 21.

As shown in fig. (21), the resultant electric field E makes an angle θ_H with X-axis. The angle θ_H is known as Hall angle.

Therefore, Hall angle,

$$\theta_H = \tan^{-1}\left(\frac{E_H}{E_x}\right) \quad \text{...(1)}$$

We have
$$E_H = \frac{V_H}{y} \quad \text{...(2)}$$

and
$$E_x = \frac{\text{Voltage drop along the length}}{\text{Length of the specimen}}$$
$$= \frac{I_x R}{l}$$
$$= \frac{I_x \times (\text{resistivity} \times l)}{l \times A}$$
$$= \frac{I_x \rho}{A}$$

where ρ is the resistivity.

So,
$$E_x = \frac{I_x \rho}{A} = \frac{J}{\sigma} \quad \text{...(3)}$$

Substituting the values of E_H and E_x from eqs. (2) and (3) respectively in eq. (1), we get

$$\theta_H = \tan^{-1}\left[\frac{V_H/y}{(J/\sigma)}\right]$$

$$= \tan^{-1}\left[\frac{B_z\, I_x/\rho\, z\, y)}{J/\sigma}\right]$$

where $$V_H = \frac{B_z\, I_x}{\rho\, z}$$

$\therefore$ $$\theta_H = \tan^{-1}\left[\frac{B_z\, J/\rho}{J/\sigma}\right]$$

$$= \tan^{-1}\left[\frac{B_z\, \sigma}{\rho}\right]$$

or $$\theta_H = \tan^{-1}(B_z\, \sigma\, R_H) \quad \left[\because\ R_H = \frac{1}{n\,e} = \frac{1}{\rho}\right] \quad ...(4)$$

or $$\theta_H = \tan^{-1}(\mu\, B_z) \quad ...(5)$$

5.15 MEASURAEMENT OF CONDUCTIVITY OF A SEMICONDUCTOR

A four probe experimental setup is shown in fig. (22). As a sample, we take a Ge or Si crystal chip with non-conducting base.

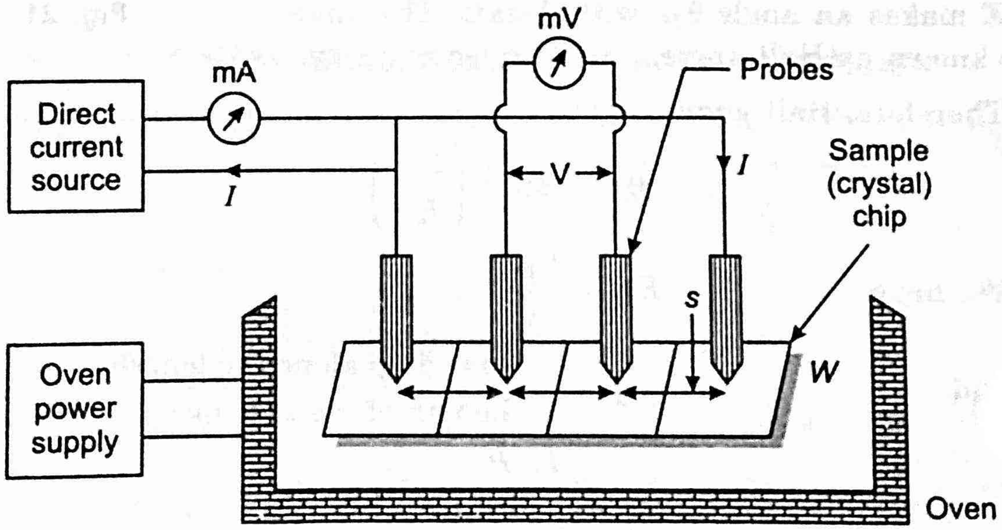

Fig. 22.

Distance between probes is, s and thickness of crystal chip is W. Then value of (W/s) is calculated.

The conductivity of crystal chip, σ, is given by

$$\sigma = \frac{f(W/s)}{\rho_o} \quad ...(1)$$

where $$\rho_o = \frac{V}{I} \times 2\pi s$$

Voltage, V is reading of millivoltmeter and I is a constant current read in milliammeter. Function $f(W/s)$ is tabulated in the Table. Plot a graph in (W/s) and $f(W/s)$ values and find value of $f(W/s)$ corresponding to (W/s) value we calculated.

(W/s)	f(W/s)
0.100	13.863
0.141	9.704
0.200	6.931
0.333	4.159
0.500	2.780
1.000	1.504
1.414	1.223
2.000	1.094
3.333	1.0228
5.000	1.0070
10.000	1.00045

Place the probe arrangement in oven and send a constant current, I. Start heating. At two different temperatures, measure two values of voltage, V, across probes. From these values, find two values of ρ_0 using eq. (1), corresponding to two different temperatures. Using these two values of ρ_0 and the value of function $f(W/s)$ found from graph, we can find the conductivity, σ, of semiconductor crystal chip.

24.16 MOBILITY

The conductivity of a semiconducting material is given by

$\sigma = ne\mu$

where μ is the mobility of the charge carrier.

But $$ne = \frac{1}{R_H}$$, from equation (x). On substitution, we get

$$\sigma = \frac{\mu}{R_H} \quad \text{... (xii)}$$

$\therefore$ Mobility , $\mu = \sigma R_H$

Knowing, the Hall coefficient R_H and the conductivity (σ) of the sample, the mobility (μ) of charge carrier can be evaluated.

S.I. unit of Hall coefficient R_H is m^3/Coulomb.

USES : (i) Determination of semiconductor type can be obtained if the Hall coefficient is positive, then it a *P*-type semiconductor. For negative value of R_H, we have *N*-type semiconductor.

(*ii*) By measuring Hall coefficient R_H, we can find electron concentration n_e and hole concentration n_p,

(*iii*) Using this method, by measuring the conductivity σ and Hall coefficient, we can measure the mobility μ.

(*iv*) We can also measure the strength of magnetic field *B* by knowing the Hall voltage, Hall coefficient and amount of current passed.

(*v*) This method can be used to determine the power flow in an electromagnetic wave.

This is done by knowing that the Hall voltage is proportional to the product of $\vec{E}$ and $\vec{\beta}$.

6.7 HALL EFFECT

Hall-Effect in semiconductors comes from motion of both electrons and holes. Edwin Hall (1855 – 1938) an American physicist in 1879 explained this effect which states that "**when a current carrying conductor is placed in a magnetic field mutually perpendicular to the direction of current, a potential difference is developed at right angles to both the magnetic and electric field.**"

Hall Effect in Semiconductors : When a current carrying semiconductor is placed in a magnetic field, the charge carriers of the semiconductor experience a force in a direction perpendicular to both the magnetic field and the current.

Hall effect gives the best experimental proof of the motion of holes.

Hall Electric field (E_y) : Let us derive an expression for the Hall electric field E_y when both electrons and holes are deflected transversely with an applied electric field E_x along *X*-axis and a transverse component of magnetic field B_z is applied along *Z*-axis.

The electron current density J_{xe} along *x*-axis is given as

$$J_{xe} = n_e \mu_e E_x$$

where n_e is the number of electrons and μ_e is the mobility of electrons.

Similarly, the expression for hole current. The net transverse current density J_y along Y-axis is density along x-axis is given by

$$J_{xh} = n_p \mu_p E_x$$

where n_p is the number of holes and μ_p is the mobility of holes.

The total current density J_x along X-axis in a semiconductor is given as

$$J = J_{xe} + J_{xh}$$

$$= e(n_e \mu_e + n_h \mu_h) E_x$$

Now, the magnetic Lorentz force acting on an electron moving with drift velocity v_e in the magnetic field B_z is equal to $-e(v_e \times \vec{B_z})$ or is equal to $e\, v_y\, B_z$. This Lorentz force is equivalent to an electric field $-v_e B_z$. We know that the drift velocity $\vec{v_e} = \mu_e \vec{E_x}$ and therefore,

$$J_{ye} = n_e\, e\, \mu_{ez}(\mu_e B_z E_x)$$

$$= n_e e^2 \mu_e^2 B_z E_x$$

Similarly hole current density along y-axis is

$$J_{yh} = n_h\, e\, \mu_h^2 B_z E_x$$

$$J_y = J_{ye} + J_{yh}$$

$$= e(n_e \mu_e^2 - n_h \mu_h^2) B_z E_x$$

To make net current along y-axis equal to zero, a Hall electric field E_y is required.

The ratio $$\frac{E_y}{E_x} = \frac{J_y}{J_x}$$

$$\therefore \quad e(n_e \mu_e + n_h \mu_h) E_y = -e(n_e \mu_e^2 - n_p \mu_h^2) B_z E_x$$

or $$E_y = \left[\frac{n_h \mu_h^2 - n_e \mu_e^2}{n_e \mu_e + n_h \mu_h}\right] B_z E_x$$

6.8 MEASUREMENT OF CONDUCTIVITY AND HALL COEFFICIENT

The Hall Coefficient R_H is defined as the ratio of the electric field induced to the product of the current density J_x and the applied magnetic field B_z. Mathematically

$$R_H = \frac{E_y}{B_Z J_x} = \frac{n_h \mu_h^2 - n_e \mu_e^2}{e(n_e \mu_e + n_h \mu_h)^2}$$

Four Probe Method : This is a simple method in which a rectangular shaped semiconductor in the form of a wafer is taken. It consists of four probes arranged linearly in a straight line at equal distances from each other. A constant current is passed through the two probes and the potential difference is measured across the middle two probes.

Experimental Setup : A rectangular specimen of a semiconductor of width b and thickness t is placed between the pole pieces of an electromagnet as shown in Fig. 6.17.

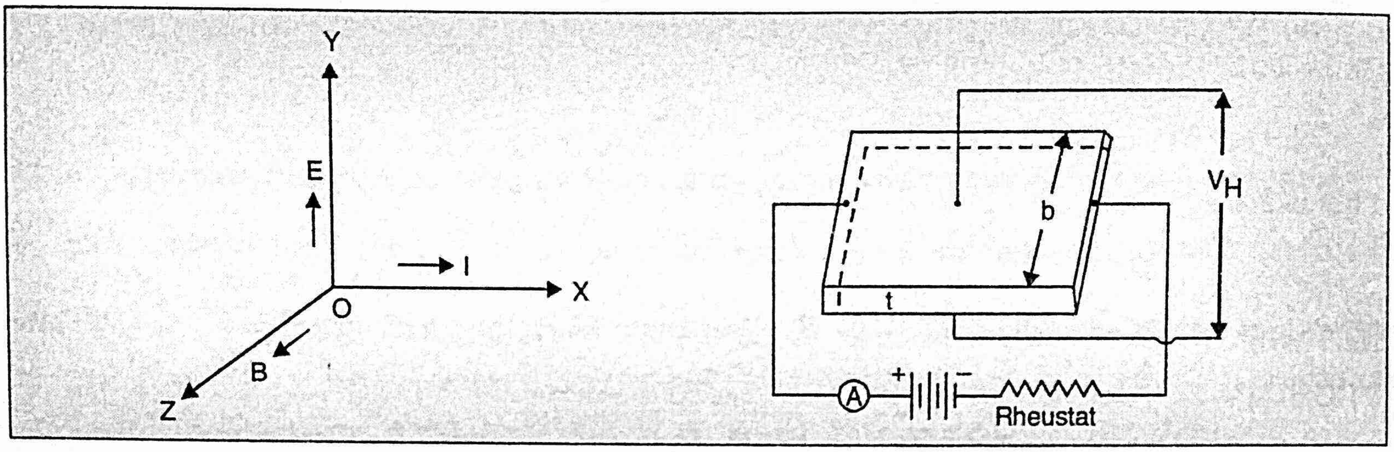

Fig. 6.17

A magnetic field $\vec{B}$ is applied along the Z-axis perpendicular to the applied electric field. A current I passes along X-axis from a battery. An ammeter A is connected in series to measure the current.

Calculations : The Hall voltage V_H is measured by placing two probes at the centre of the bottom and top surfaces of semiconductor sample (Fig. 6.17).

$$V_H = E_H t$$

where Hall electric field $E_H = R_H J B_Z$

So that

$$V_H = R_H J B_Z t$$

Now, we know that the current density J is given as

$$J = \frac{I}{\text{Area}}$$

$$= \frac{I}{tb} \qquad (\because \text{Area} = \text{thickness} \times \text{breadth})$$

we know that $J = \sigma E_H$

or conductivity, $\sigma = = \frac{J}{E_H} = \frac{I}{\tau b E_H}$

and $R_H = \frac{V_H \, b}{I \, B_Z}$

S.I. unit of Hall coefficient R_H is m^3/Coulomb.